1. A laser with constant amplitude $E_0$ and frequency $\omega_0$ is incident on a polariser which is rotating with angular frequency $\omega_{pol}$. At $t = 0$, $E_0$ is parallel with the transmission axis of the polariser. Show that the time varying intensity after the polariser is given, up to some constants, by:

$$I(t) = E_0^2 \cos^2(\omega_{pol} t)$$

Calculate the second order correlation function $g^{(2)}(\tau)$ and show that it satisfies the classical limits of $g^{(2)}(0) \geq 1$ and $g^{(2)}(0) \geq g^{(2)}(\tau)$. Sketch $g^{(2)}(\tau)$ as a function of $\tau$ and find the time interval ($\Delta T$) between maximally correlated events. How many degrees did the polariser move during $\Delta T$?

Transmissivity of polariser is given by $\cos(\alpha)$, where $\alpha$ is the angle between the transmission axis and the electric field vector. The polariser is turning with angular frequency $\omega_{pol}$, hence the time-dependent electric field after the polariser is given by:

$$E(t) = E_0 \cos(\omega_{pol} t)$$

As the intensity is proportional to the square of the electric field, we have:

$$I(t) = E_0^2 \cos^2(\omega_{pol} t)$$

The second order correlation function is defined as:

$$g^{(2)}(\tau) = \frac{\langle I(t)I(t+\tau) \rangle}{\langle I(t) \rangle \langle I(t+\tau) \rangle}$$

Which yields:

$$g^{(2)}(\tau) = \frac{E_0^4 \langle \cos^2(\omega_{pol} t) \cos^2(\omega_{pol} (t+\tau)) \rangle}{E_0^4 \langle \cos^2(\omega_{pol} t) \rangle \langle \cos^2(\omega_{pol} (t+\tau)) \rangle}$$

for the light field above. The long time average for $\langle \cos^2(\omega_{pol} t) \rangle = \langle \frac{1 + \cos(2\omega_{pol} t)}{2} \rangle = \frac{1}{2}$, since the time average of $\cos(2\omega_{pol} t)$ is $\frac{1}{T} \int_0^T \cos(2\omega_{pol} t) \, dt = 0$ for large $T$. Similarly $\langle \cos^2(\omega_{pol} (t + \tau)) \rangle = \frac{1}{2}$. The numerator $E_0^4 \langle \cos^2(\omega_{pol} t) \cos^2(\omega_{pol} (t + \tau)) \rangle$ can be expanded to become:

$\frac{E_0^4}{4} \langle (1 + \cos(2\omega_{pol} t) + \cos(2\omega_{pol} (t + \tau)) + \cos(2\omega_{pol} t) \cos(2\omega_{pol} (t + \tau))) \rangle$.

The second and third term are zero, by the argument above. Using a trigonometric identity we can rewrite $\langle \cos(2\omega_{pol} t) \cos(2\omega_{pol} (t + \tau)) \rangle$ as $\frac{1}{2} \langle \cos(2\omega_{pol} \tau) + \cos(2\omega_{pol}(2t + \tau)) \rangle$, which yields $\frac{1}{2} \cos(2\omega_{pol} \tau)$ after the time average. Hence the numerator becomes: $\frac{E_0^4}{4} (1 + \frac{1}{2} \cos(2\omega_{pol} \tau))$.

Finally we get for the correlation function:
\[ g^{(2)}(\tau) = \frac{E_0^4}{4} \left(1 + \frac{1}{2} \cos(2 \omega_{\text{pol}} \tau)\right) = 1 + \frac{1}{2} \cos(2 \omega_{\text{pol}} \tau). \]  

At \( \tau = 0 \), we have \( g^{(2)}(0) = \frac{3}{4} > 1 \) and since the cosine can only vary between -1 and 1 we also have \( g^{(2)}(0) \geq g^{(2)}(\tau) \).

\[ \Delta T, \text{ the time between maximally correlated events is given by } \Delta T = \pi \omega_{\text{pol}}, \text{ which corresponds to a rotation of 180 degree by the polariser.} \]

2. A source emits a train of single photons with regular time intervals between them. Sketch \( g^{(2)}(\tau) \) that would be expected if:

(a) the time interval between photons is much larger than the response time \( \tau_D \) of the detector.
(b) the time interval between photons is much smaller than \( \tau_D \).
When the detector response time is small compared to the photon period then events are only registered close to the actual time of the photons (spikes). In that case we also have $g^{(2)}(0) = 0$, as only one photon is present per counting bin.

If the detector response time is larger than the photon period, then there is an uncertainty as to when the photon is detected (broadening of peaks). This leads to a nonzero value of $g^{(2)}(0)$, since two consecutive photons can now fall into the central timing bin.