Solutions for Exercise set III

1. Exercise 5.6 form course book "Quantum Optics".

   (a) the mean photon number per pulse:
   The mean photon number ($\bar{n}$) per pulse is: $\bar{n} = \frac{E_{\text{pulse}}}{\hbar \omega}$, where $E_{\text{pulse}}$ is the energy per pulse.
   $E_{\text{pulse}} = \frac{P}{N_{\text{pulse}}}$, where $P$ is the optical power and $N_{\text{pulse}}$ is the number of pulses per second.
   So we have, $\bar{n} = \frac{P}{\hbar \omega N_{\text{pulse}}} = \frac{P \lambda}{\hbar 2 \pi c N_{\text{pulse}}} = 4.0 \times 10^7$

   (b) the standard deviation per pulse:
   The photon statistics of a laser is Poissonian, so $\Delta n = \sqrt{\bar{n}} = 6.4 \times 10^3$.

2. Exercise 5.7 form course book "Quantum Optics". The laser described in the previous question is attenuated by a factor $10^9$. For the attenuated beam calculate:

   (a) the mean photon number per pulse:
   $\bar{n} = \frac{n_{\text{old}}}{10^9} = 0.04$

   (b) the the fraction of pulses containing one photon:
   From Poission distribution we have: $P(1) = \bar{n} e^{-\bar{n}} = 0.04 e^{-0.04} = 0.038$

   (c) the the fraction of pulses containing more than one photon:
   $P(n > 1) = 1 - P(0) - P(1) = 1 - e^{-\bar{n}} - \bar{n} e^{-\bar{n}} = 7.8 \times 10^{-4}$. One pulse in every 1284 pulses has more than 1 photon.

3. Exercise 5.8 form course book "Quantum Optics".
   Average photon number impinging on detector is: $\bar{n} = \Phi T = 10^4$, where $\Phi$ is the photon flux and $T$ is the time interval. Average detected count rate $\bar{N} = \eta \bar{n} = 2 \times 10^3$. 

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(a) the light has Poissonian statistics:
Mean photon number: $\bar{N} = 2 \times 10^3$.
Standard deviation: From equation 5.56 we have: $(\Delta \bar{N})^2 = \eta \bar{n} = \bar{N} \Rightarrow \Delta \bar{N} = \sqrt{\bar{N}} = 44.7$.

(b) the light has super-Poissonian statistics with $\Delta n = 2 \times \Delta n_{Poissionian}$:
Mean photon number: $\bar{N} = 2 \times 10^3$.
Standard deviation: $(\Delta \bar{N})^2 = \eta^2 4\bar{n} + \eta(1-\eta)\bar{n} = 0.32\bar{n} = 3200. \Rightarrow \Delta \bar{N} = 56.6$.

(c) the light is a photon number state:
Mean photon number: $\bar{N} = 2 \times 10^3$.
Standard deviation: $(\Delta \bar{N})^2 = \eta(1-\eta)\bar{n} = 0.16\bar{n} = 1600. \Rightarrow \Delta \bar{N} = 40.0$.

4. Exercise 5.13 from course book "Quantum Optics". An LED emitting light at 800 nm is driven by a 9 V battery through a resistor with $R = 1000\Omega$. The LED has a quantum efficiency of 40% and 80% of the photons emitted are focused onto a photodiode detector with a quantum efficiency of 90%.

(a) Calculate the average drive current, given that the voltage drop across the LED is approximately equivalent to the photon energy in eV in normal operating conditions:
Photon energy is $h \frac{2\pi c}{\lambda} = 1.55$ eV. Voltage drop at LED is 1.55 V. Voltage across resistor is $9 - 1.55 = 7.45$ V. Current is $\frac{U}{R} = 7.45$ mA.

(b) Calculate the Fano factor of the drive current for $T = 293$ K:
$F_{drive} = \frac{(\Delta i)^2}{(\Delta i)^2_{shot-noise}} = \frac{4k_b T f}{e^2 R c} = 7.0 \times 10^{-3}$.

(c) Calculate the average photocurrent in the detection circuit:
Average photocurrent is:
$<i_{photo}> = \eta_{DET} e \Phi = \eta_{DET} e \frac{<I_{drive}>}{e} \eta_{LED} \eta_{optics} = 2.15$ mA.

(d) Calculate the Fano factor of the photocurrent:
$F_{photo} = \eta_{TOT} F_{drive} + (1 - \eta_{TOT}) = 0.714$.

(e) Compare the photocurrent noise power in a 50Ω load resistor with the shot noise level for a bandwidth of 10 kHz: Shot-noise power:
$P_{shot-noise} = 2 e R L \Delta f <i> = 0.34$ fW. $F_{photo} = \frac{P_{measured}}{P_{photo}} = 0.714. \Rightarrow P_{measured} = 0.25$ fW.